Test 4
Thursday 28 June 2018

Sets, Probability, Counting and Trigonometric Identities

MATHEMATICS
METHODS 1&2

Student Name:_____

Time allowed for this assessment

ALL Calculator-assumed
Working time for this section: 50 minutes   Total marks available: 49
Formulae Sheet allowed.
Classpad assumed.
1 A4 page (single sided) of notes allowed.

1. [1, 1, 1, 2 = 5 marks]

   Given that $U = \{x \mid 40 \leq x \leq 60\}\, where\, x\, is\, an\, integer$, $A = \{41, 43, 55, 58\}$,

   $B = \{52, 54, 55, 56\} \wedge C = \{41, 43, 56, 60\}$

   a) Find $\{U\}$

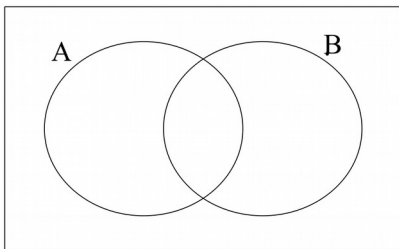   b) Find $A \cup B$

   c) Determine $n(C \cap \acute{B})$

   d) Determine $P(A \cup \acute{B} \cup C)$
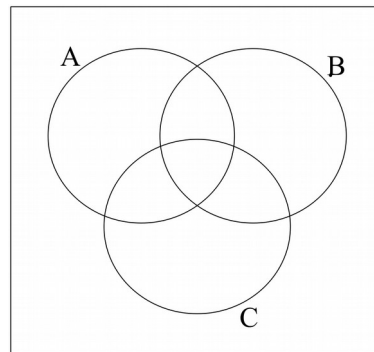
2. [1, 1 = 2 marks]

   In the Venn diagrams below, shade the region representing:

   a) $A' \cap B'$

   b) $(B \cup C) \cap \acute{A}$

3.   [1, 1, 3 = 5 marks]

Evaluate each of the following, **show your working for full marks to be awarded**:

(a) $6!$

(b) $^6P_4$

(c) $\begin{pmatrix} 6 \\ 3 \end{pmatrix} \times 3!$

4.   [4 marks]

Expand and simplify completely using an appropriate method that you have learnt:

$(2-x)^5$

5. [1, 2 = 3 marks]

Consider the expansion for: $(x+3y)^{11}$ in descending powers of x.

(a) How many terms are there in this expansion?

(b) Find the 3rd term in this expansion using Combination Theory $\begin{pmatrix} n \\ r \end{pmatrix}$

4

6. [2, 1, 1, 1, 2, 2 = 9 marks]

An analysis of the 210 students in their final year of school determined that 35 chose to study Physics, 45 chose to study Chemistry and 151 chose neither of these subjects.

(a) Complete the Venn diagram below using the information given.
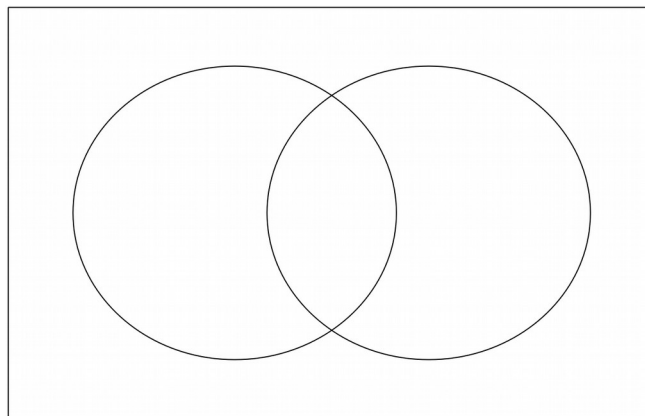
(b) Determine the number of students who chose to study both Physics and Chemistry.

(c) Determine the probability that a randomly selected student chose to study

(i) Chemistry.

(ii) Physics but did not choose Chemistry.

(iii) Chemistry given that they chose to study Physics.

(d) Is there any indication that choosing to study Chemistry is independent of choosing to study Physics? Justify your answer.

5

7. [1, 2, 3, 3 = 9 marks]

Given $P(A)=m+0.2$ and $P(B)=m+0.3$ and $P(A \cap B)=m$, calculate the value of m if:

(a)   A and B are mutually exclusive

(b) $P(A \cup B)=0.6$

(c)   A and B are independent events

(d) $P(A \vee B)=0.5$

8. [1, 1, 2 = 4 marks]

A committee consisting of 10 senior members and 12 junior members has decided to select five of its members to form a subcommittee.

(a) Determine the number of different subcommittee combinations that can be selected.

(b) Determine the number of different subcommittees that can be selected that contain only senior members.

(c) Determine the probability that a randomly chosen subcommittee contains at least one junior member.

7

9. [4, 4 = 8 marks]

(a) If A and B are acute angles, with $\sin(A) = \dfrac{3}{5}$ and $\tan(B) = \dfrac{12}{5}$ show that

$\cos(A+B) = \dfrac{-16}{65}$.

(b) Solve the equation $\sqrt{3}\cos\left(x - \dfrac{\pi}{2}\right) = \cos(x)$ for $0 \le x \le 2\pi$.

8